Problem 4.11

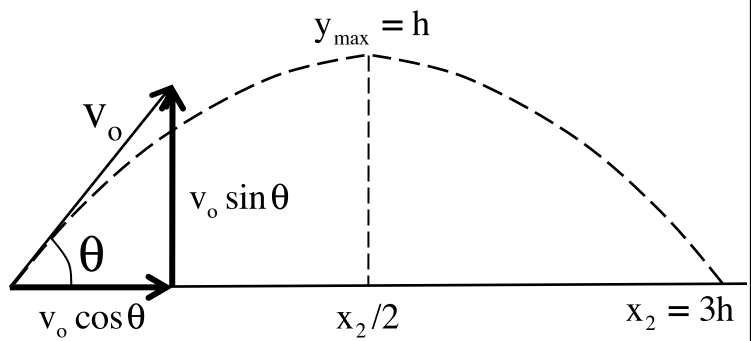
Fun stuff! This is one of those problems where you have to range around, looking at all the equations available and see which ones will do the job for you. Acknowledging that your first try might not do the trick, at the very least we can note what we've been told. Specifically, that:

$$x_2 = 3y_2 = 3h$$

And as always, the velocity in the y-direction at the top is zero (which is not to say the velocity magnitude is zero--there is still a component in the x-direction), or:

$$v_{y_{max}} = 0.$$

The sketch (always a good idea if you have the time) is presented to the right.



What we need are relationships that are true. Taking the time interval *t* to be the time to touchdown, we can write:

$$\cancelto{3h}{x_2} = \cancelto{0}{x_1} + (v_o \cos\theta)\Delta t + \frac{1}{2}\cancelto{0}{a_x}(\Delta t_{midpoint})^2$$

$$\Rightarrow \quad 3h = (v_o \cos\theta)t$$

$$\Rightarrow \quad t = \frac{3h}{v_o \cos\theta}$$

and:

$$\cancelto{0}{y_2} = \cancelto{0}{y_1} + v_{1,y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$\Rightarrow \quad (v_o \sin\theta)\cancel{t} = \frac{1}{2}g(t)^{\cancel{2}}$$

$$\Rightarrow \quad \frac{2v_o \sin\theta}{g} = t$$

Equating the relationships for *t* yields:

$$t = \frac{3h}{v_o \cos\theta} = \frac{2v_o \sin\theta}{g}$$

$$\Rightarrow \quad (v_o)^2 = \frac{3hg}{2\sin\theta\cos\theta}$$

The "maximum height" relationship yields:

$$(\overset{0}{\cancel{v_{y,max}}})^2 = (v_{y,1})^2 + 2a_y(y_{max} - y_1)$$

$$\Rightarrow \quad 0 = (v_o \sin\theta)^2 + 2(-g)y_{max}$$

$$\Rightarrow \quad (v_o \sin\theta)^2 = 2gh$$

$$\Rightarrow \quad v_o^{\ 2} = \frac{2gh}{(\sin\theta)^2}$$

Combining the velocity terms yields:

$$(v_o)^2 = \frac{3h\cancel{g}}{2\sin\theta\cos\theta} = \frac{2\cancel{g}h}{(\sin\theta)^2}$$

$$\Rightarrow \quad \frac{(\sin\theta)^{\cancel{2}}}{\cancel{\sin\theta}\cos\theta} = \frac{4}{3}$$

$$\Rightarrow \quad \tan\theta = \frac{4}{3} \quad \Rightarrow \quad \theta = 53.1^o$$

Like I said, obscure: